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**RESEARCH ARTICLE**

# Some Applications of Semi Group Theory

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**ABSTRACT**

This paper deals with some applications of semi groups in the various fields. Consider such applications Partial differential equations; automate theory, biology, formal language, sociology, network analogy, etc.

**Keywords:** Semigroup**,** Partial differential equation, Sociology, Kinship system.

## INTRODUCTION

In mathematics, a semigroup is an algebraic structure consisting of a set together with an associative binary operation. The binary operation of a semigroup is most often denoted multiplicatively : '*a.b*', or simply '*ab*', denotes the result of applying the semigroup operation to the ordered pair (a,b). Associativity is formally expressed as that $(a.b).c = a.(b.c)$ for all *a,b* and *c* in the semigroup.The theory of finite semigroups has been of particular importance in theoretical computer science since the 1950s due to the natural link between finite semigroups and finite automata via the syntactic monoid. In probability theory, semigroups are associated with Markov processes. In other areas of applied mathematics, semigroups are fundamental models for linear time-invariant systems. In partial differential equations, a semigroup is associated to any equation whose spatial evolution is independent of time. The areas of applications of such as Partial differential equations, formal languages and the software use the language of recent algebra in terms of Boolean logic, semigroups et al..But also parts of other area like biology, biochemistry and sociology.

The theory of automata has its origins within the work by Turing (Shannon 1984 and Heriken 1994). Turing developed the theoretical concept of what's now called Turingmachines, so as to offer computability a more concrete and precise meaning. Many various parts of pure mathematicians are used as tools like abstract algebra, universal algebra, lattice theory, graph theory and therefore the theory of algorithms.The beginning of the of formal languages can be traced Chomsky, who introduced the concept of context-free language so as to model natural languages in 1957. Semigroups are often utilized in biology to explain certain aspets within the crossing of organisms, in genetics and in consideration of metabolisms. The expansion of plants are often described algebraically in Hermann and Rosenberg (1957). Language theory is employed in cell – development problems, as introduced by Lindenmayes (1968), Hermann and Rosendalg (1975). Sociology includes the study of human interactive behavior in group situations, especially in underlying structures of societies. Such structures are often revealed by mathematical analysis. This means how algebraic techniques could also be introduced into studies of this type.

## Applications of Semi groups

### Semi groups - partial differential equations

Semigroup theories are often wont to study some problems wthin the field of partial differential equations. Roughly speaking, the semigroup approach is to regard a time-dependent partial differential equation as an ordinary differential equation on a function space.

For example, consider the subsequent initial/boundary value problem for the heat equation on the spatial interval (0, 1) ⊂ **R** and times $t \geq 0$:

$$\begin{cases} \partial_t u(t,x) = \partial_t^2 u(t,x), & x \in (0,1), t > 0; \\ u(t,x) = o, & x \in \{0,1\}, t > 0; \\ u(t,x) = u_0(x), & x \in (0,1), t = 0. \end{cases}$$

Let $X = L^2((0,1)R)$ be the $L^p$ of square-integrable real-valued functions with domain the interval (0, 1) and let $A$ be the second-derivative operator with domain
$D(A) = \{u \in H^2((0,1);R)/u(0) = u(1) = 0\}$,

Where $H^2$ is a Sobolev space. Then the above initial/boundary value problems are often interpreted as an initial value problem for an ordinary differential equation on the space $X$:

$$\begin{cases} \dot{u}(t) = Au(t); \\ u(0) = u_0. \end{cases}$$

On a heuristic level, the answer to the present problem "ought" to be $u(t) = e^{(tA)}u_{0.}$ however, for a rigorous treatment, a meaning must tend to the exponential of $tA$. As a function of $t$, $e^{(tA)}$ is a semigroup of operators from $X$ to itself, taking the initial state $u_{0.}$ at time $t = 0$ to the state $u(t) = e^{(tA)}u_{0.}$ at time $t$. The operator $A$ is said to be the infinitesimal generator of the semigroup.

### Semi groups - Biology

Semi groups are often utilized in biology to explain certain aspects in the crossing of organisms, in genetics, and in consideration of metabolisms.

### Example

In breeding a strain of cattle, which can be white or red monochromatic or spotted, it's known that white is dominant and red receive which monochromatic is dominant over spotted. Thus there are four possible sorts of cattle in this herd.

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x = white monochromatic
y = white spotted
z = Red monochromatic
w = Red spotted.
Due to dominance, in crossing a white spotted one with a red monochromatic one, we expect a white monochromatic one.
This will be symbolized by
$y\ o\ z = x$. The operation 'o 'can be studied for all possible pairs to get the table.

| *o* | x | y | z | w |
|---|---|---|---|---|
| x | x | x | x | x |
| y | x | y | x | y |
| z | x | x | z | z |
| w | x | y | z | w |

Then $A = \{x, y\ z, w\}$ is a semi group with identity element w.

**Semi groups – Sociology**

Sociology includes the study of human interactive behavior in group situations, especially in underlying structures of societies. Such structures are often revealed by mathematical analysis. This means how algebraic techniques could also be introduced into studies of this type.

(S(A), *) is called the relation semi group on A. The operation 'o' is called the relation product, where A is a monoid.
A kinship system may be a semi group R = {X, A} where A is a relation on X, which express equality of kinship relationships.

**Example**
Let the Kinship system R = {X, A} be defined by
X = {U = ("is mother of"), V = ("is son of")},
R = {(UU, U), (UV, VU), (VV, V)}. Let x,y,z be the equivalence class of U, V and UV respectively. Now "o" is given by

| *o* | x | y | z |
|---|---|---|---|
| x | x | z | z |
| y | z | y | z |
| z | z | z | z |

Then (R,o) is a semi group.

## CONCLUSION

We've seen different areas of applications of semi groups. We identified some examples in biology, sociology and partial differential equations etc. Further we would like to review some more structures of semi groups which can find applications in several areas.

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